Student Number:

Teacher:

St George Girls High School

Mathematics Extension 1

2022 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided

  For questions in **Section II:**
  - Answer the questions in the booklets provided
  - Start each question in a new writing booklet
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 70

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 –12)

- Attempt Questions 11–16
- Allow about 1 hour and 45 minutes for this section

Q1-10	/10
Q11	/10
Q12	/10
Q13	/10
Q14	/10
Q15	/10
Q16	/10
TOTAL	**/70**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet provided for Questions 1 to 10.

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1. Which of the following is the derivative of $arc\sin\left(\frac{x}{3}\right)$?

A. $\frac{1}{3\sqrt{1-x^2}}$

B. $\frac{1}{\sqrt{9-x^2}}$

C. $\frac{1}{3\sqrt{1-9x^2}}$

D. $-\frac{1}{3}\cos\left(\frac{x}{3}\right)$

2. Given that $f(x) = \ln(2-x)$, what is the range of $f^{-1}(x)$?

A. $[0,2]$

B. $[0,2)$

C. $[2,\infty)$

D. $(-\infty,2)$

3. Find $\int \cos 5x \sin 3x \, dx$.

A. $-\frac{1}{15}\sin 5x \cos 3x + C$

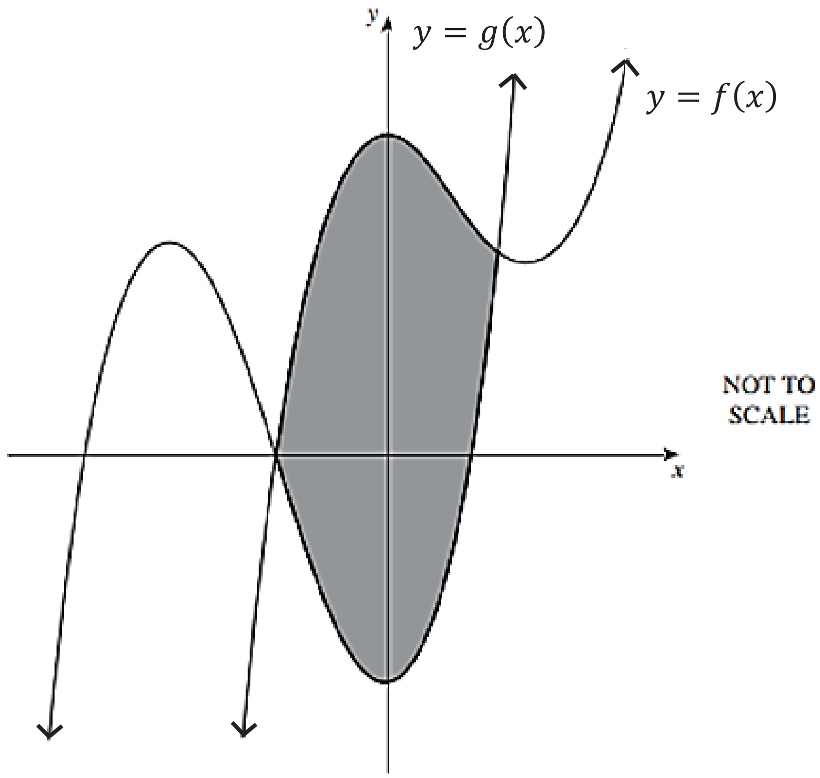
B. $\frac{1}{16}(4\cos 2x - \cos 8x) + C$

C. $\frac{1}{2}(\sin 8x - 4\sin 2x) + C$

D. $\frac{1}{2}(\sin 8x + \sin 2x) + C$

4. Consider two curves with the equation $f(x) = x^3 - 2x^2 + 3$ and $g(x) = x^3 + 3x^2 - 2$.

The diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



Which of the following gives the correct expression for the shaded area between the two curves?

A. $\int_{-2}^{3} (-5x^2 + 5)\, dx$

B. $\int_{-2}^{3} (5x^2 - 5)\, dx$

C. $\int_{-1}^{1} (5x^2 - 5)\, dx$

D. $\int_{-1}^{1} (-5x^2 + 5)\, dx$

5. Consider the vectors $\underset{\sim}{p} = \binom{t-8}{6}$ and $\underset{\sim}{q} = \binom{3}{2t}$.

What are the possible values of t so that $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are parallel?

A. $-3, -11$

B. $-1, 9$

C. $1, -9$

D. $3, 11$

6. A Biology class consists of 10 girls and 15 boys.
In how many ways can a group of three boys and two girls be chosen from this class to work on an investigation project?

A. 250

B. 900

C. 12 600

D. 20 475

7. Find the volume generated when $y = \sin x$ between $x = 0$ and $x = \dfrac{\pi}{3}$ is rotated around the $x-$axis.

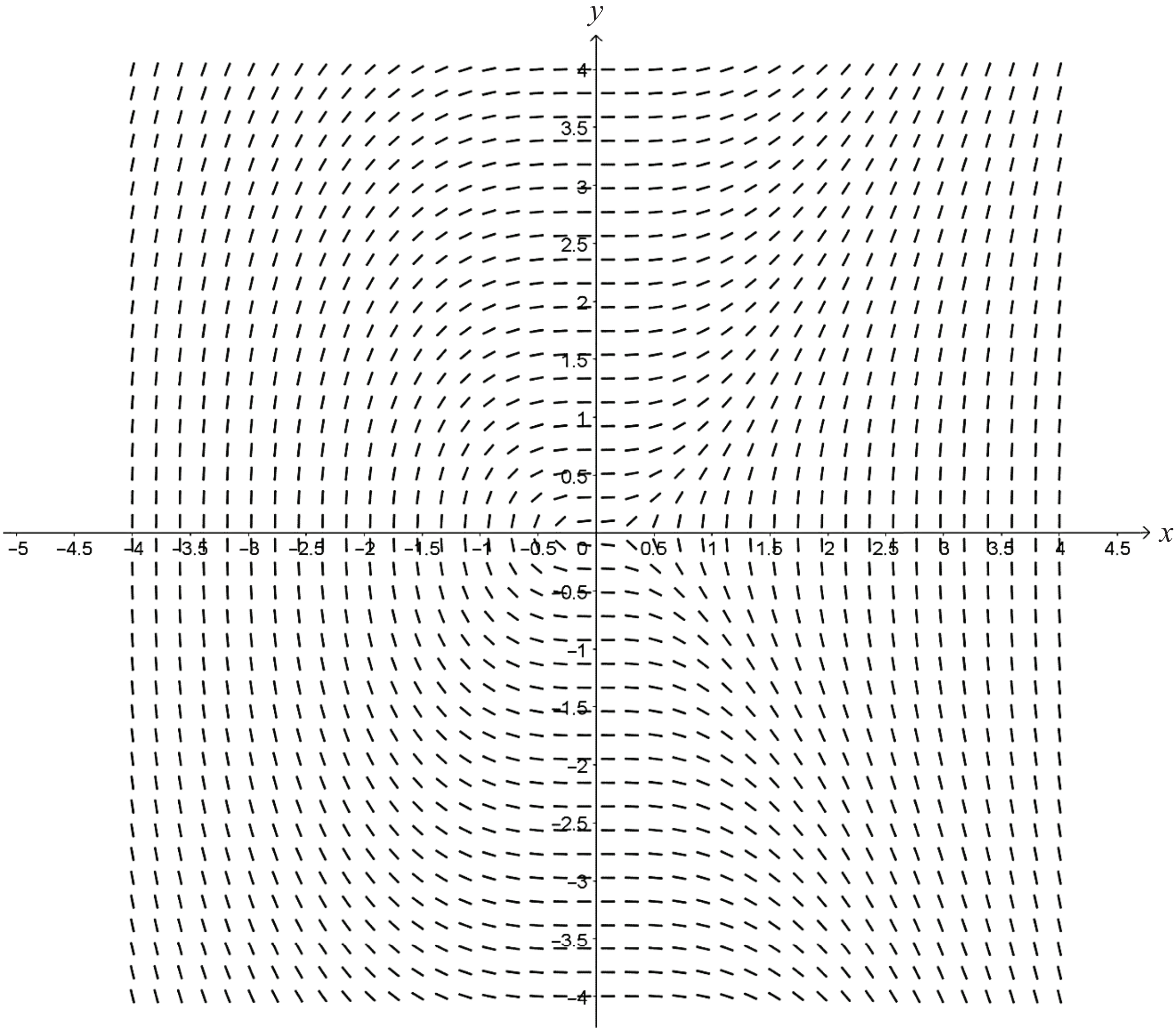
A. $\dfrac{\pi^2}{6} - \dfrac{3}{8}$ cubic units.

B. $\dfrac{\pi^2}{6} - \dfrac{\sqrt{3}\pi}{8}$ cubic units.

C. $\dfrac{\pi^2}{3} - \dfrac{3\pi}{4}$ cubic units.

D. $\dfrac{\pi^2}{6} - \dfrac{\sqrt{3}\pi}{4}$ cubic units.

8. The slope field for a differential equation is shown below.



Which of the following could be the differential equation represented?

A. $\dfrac{dy}{dx} = x^2 y$

B. $\dfrac{dy}{dx} = \dfrac{y^2}{x+y}$

C. $\dfrac{dy}{dx} = \dfrac{y}{x^2}$

D. $\dfrac{dy}{dx} = \dfrac{x^2}{y}$

9. The temperature T of a turkey placed in an oven is given by $T = 185 - Pe^{-kt}$, where P and k are positive constants and t is the time in hours.

Given that after 2 hours the rate of increase of temperature is 0.76 of the initial rate of increase of temperature, what is the value of k correct to 3 decimal places?

A. $k = 0.162$

B. $k = 0.178$

C. $k = 0.137$

D. $k = 0.014$

10. In a car yard there are 80 cars of different colours. There are 12 black cars, 11 red cars and the remaining cars are white, blue, and grey.

The number of white cars is more than the number of blue cars and grey cars.

What is the least number of white cars in the car yard?

A. 21

B. 23

C. 19

D. 20

END OF SECTION I

Section II
60 marks
Attempt Questions 11 – 16
Allow about 1 hour and 45 minutes for this section

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \cos^2\left(\frac{x}{6}\right)\, dx$. 2

(b) Find the coefficient of x^9 in the expansion of $(1+2x)\,(2+x)^{12}$. 3

(c) In how many ways can the word GEOMETRY be arranged in a straight line if the vowels must occupy the 2^{nd}, 4^{th} and 6^{th} position. 2

(d) Solve the inequality $\frac{x^2}{x-2} \geq -1$. 3

Question 12 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) If $\overrightarrow{OA} = \binom{-1}{3}$ and $\overrightarrow{OB} = \binom{2}{5}$, find $\overrightarrow{AB}$. 1

(b) Use the substitution $t = u^2 - 1$ to evaluate $\int_0^1 \frac{t}{\sqrt{1+t}}\,dt$. 3

(c) The polynomial $P(x) = x^3 + ax^2 + bx - 12$ has a double root at $x = 2$.

Find the values of a and b. 3

(d) By expressing $\sqrt{3}\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$,

solve the equation $\sqrt{3}\sin\theta - \cos\theta - 1 = 0$ for $[0\,, 2\pi]$. 3

Question 13 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the differential equation $\frac{dy}{dx} = e^x \cos^2 y$, to give an equation for y in terms of x, given that $y(2) = 0$. 3

(b) The volume of a cube is expanding at the constant rate of $5\text{mm}^3/\text{s}$.

At what rate is the surface area of the cube increasing when the side length of the cube is 60cm? 3

(c) Consider the curve $y = \sin^{-1}\left(\frac{1}{x}\right)$.

(i) Show that $y' = \frac{-1}{x\sqrt{x^2-1}}$, for $x > 0$. 2

(ii) Find the equation of the tangent to the curve $y = \sin^{-1}\left(\frac{1}{x}\right)$ at the point where $x = 2$. 2

Question 14 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use mathematical induction to prove that $10^n + 3 \times 4^{n+2} + 5$ is divisible by 9, for all positive integers $n \geq 1$. 3

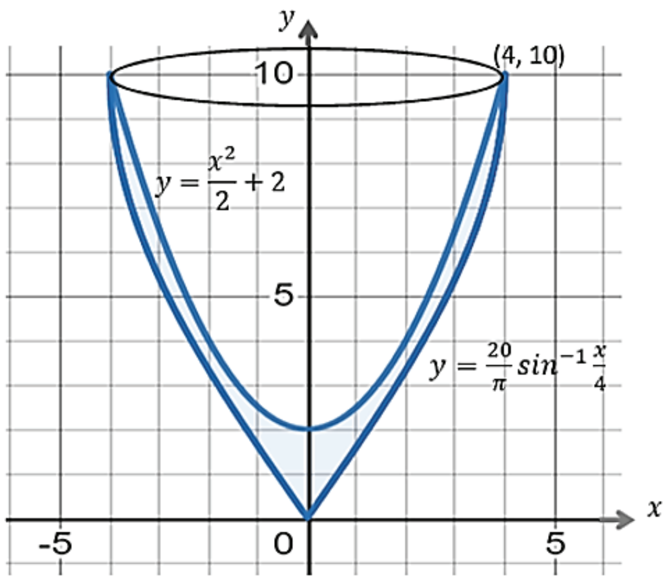
(b) Evaluate $\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$ using the substitution $x = \cos\theta$. 3

(c) A carpenter is using a lathe to create part of a small wooden bowl 10cm tall and 8cm wide, as shown below.

The shape formed is modelled by rotating the region between the curves

$y = \dfrac{x^2}{2} + 2$ and $y = \dfrac{20}{\pi}\sin^{-1}\dfrac{x}{4}$ for $0 \leq x \leq 4$, about the y-axis.

Calculate the exact volume of the timber which makes up this part of the bowl. 4

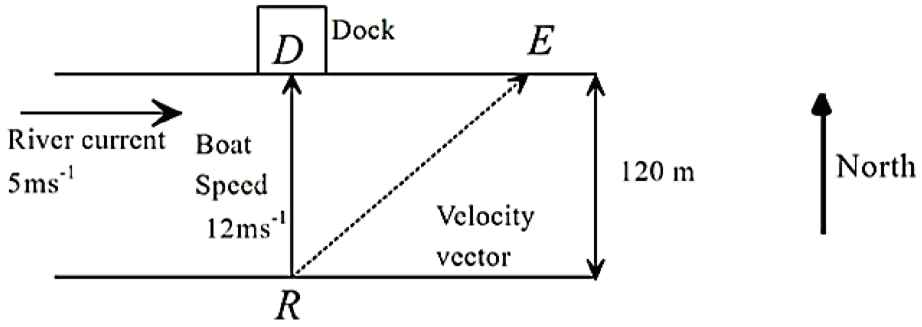


Question 15 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the equation $\sin 2x = \tan x$ for $0 \le x \le \pi$. 4

(b) Rylie has a boat which moves at a top speed of 12 ms^{-1} in still water.

From point R, he wants to go due north to point D on the opposite side of the river, as shown in the diagram below.



Today the current in the river is flowing at 5 ms^{-1}. From R, he steers the boat due north toward D at top speed. Due to the current, he drifts down the river and arrives at point E.

(i) Taking R as the origin, write down Rylie's velocity vector in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$ and find the magnitude of this vector. 2

(ii) What is the bearing of Rylie's velocity vector and how far does he travel from R to E? 2

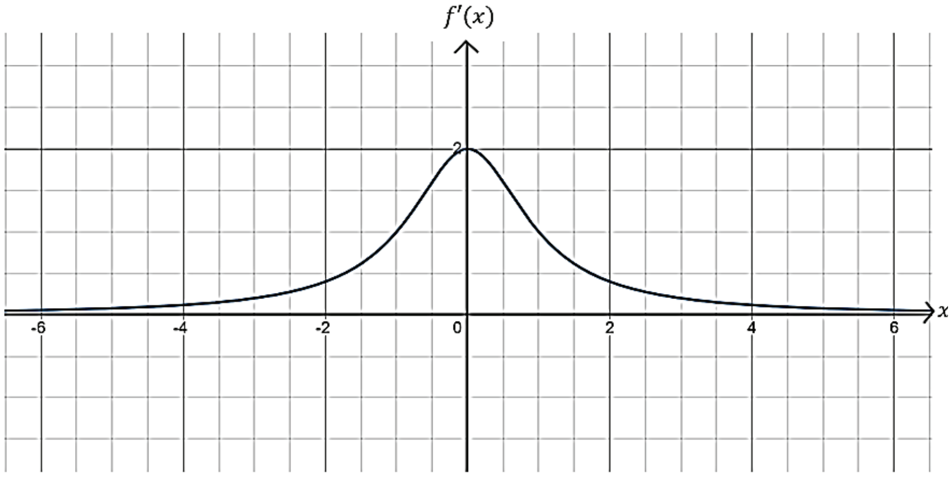
(iii) On what bearing should Rylie have pointed the boat, so that he arrived at D, with the boat travelling at its top speed? 2

Question 16 (10 marks) Use a SEPARATE writing booklet. **Marks**

(a) The rate of change of a population P can be modelled by the differential equation $\frac{dP}{dt} = kP(1 - \frac{P}{M})$ where t is the time in years, k is the proportion constant and M is a constant known as the carrying capacity (the maximum number that the population can reach).

Using the fact that $\frac{M}{P(M-P)} = \frac{1}{P} + \frac{1}{M-P}$, solve the differential equation. 4

(b) The graph below shows the **derivative** of $f(x) = 2\tan^{-1}x$.



(i) At what point does $f(x) = 2\tan^{-1}x$ have its greatest slope? 1

(ii) Write an integral that represents the area in the first quadrant bounded by the curve $y = f'(x)$, the x-axis and $x = k$, where $k > 0$. 1

(iii) By considering the limit as $k \to \infty$, determine the **total** area bounded by the curve $y = f'(x)$ and the x-axis. 2

(iv) On the separate graph provided, sketch the graph of $y = \frac{1}{f(x)}$. 2

END OF EXAMINATION

EXTENSION 1 TRIAL SOLUTIONS 2022

1. $y = \sin^{-1}\left(\frac{x}{3}\right)$ $\quad f(x) = \frac{1}{3}x$

$f'(x) = \frac{1}{3}$

$$\frac{dy}{dx} = \frac{1/3}{\sqrt{1-(\frac{1}{3}x)^2}}$$

$$= \frac{1}{3\sqrt{1-\frac{1}{9}x^2}}$$

$$= \frac{1}{3\sqrt{1/9\,(9-x^2)}}$$

$$= \frac{1}{\sqrt{9-x^2}}$$ (B)

2. Domain of $f(x)$: $2-x>0$

$x<2$

$\therefore$ Range of $f^{-1}(x)$ is $y<2$

$(-\infty, 2)$ (D)

3. $\int \cos 5x \sin 3x \, dx$ $\qquad \cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\therefore \cos 5x \sin 3x = \frac{1}{2}[\sin 8x - \sin 2x]$

$= \frac{1}{2}\int (\sin 8x - \sin 2x)\, dx$

$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x\right] + c$

$= -\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + c$

$= \frac{1}{16}(-\cos 8x + 4\cos 2x) + c$

$= \frac{1}{16}(4\cos 2x - \cos 8x) + c$ (B)

4. $x^3 - 2x^2 + 3 = x^3 + 3x^2 - 2$

$5x^2 = 5$

$x^2 = 1$

$x = \pm 1$

$f(x) - g(x)$

$= x^3 - 2x^2 + 3 - x^3 - 3x^2 + 2$

$= -5x^2 + 5$

$$A = \int_{-1}^{1} (-5x^2 + 5)\, dx$$ (D)

5. For $\underset{\sim}{p}$ and $\underset{\sim}{q}$ to be parallel

$\underset{\sim}{p} = \lambda \underset{\sim}{q}$ where λ is some scalar

$\begin{pmatrix} t-8 \\ 6 \end{pmatrix} = k\begin{pmatrix} 3 \\ 2t \end{pmatrix}$

$t-8 = 3k$ $\qquad 6 = 2tk$

$k = \frac{3}{t}$

Sub $k = \frac{3}{t}$ into $t-8 = 3k$

$t - 8 = 3\left(\frac{3}{t}\right)$

$t^2 - 8t = 9$

$t^2 - 8t - 9 = 0$

$(t-9)(t+1) = 0$

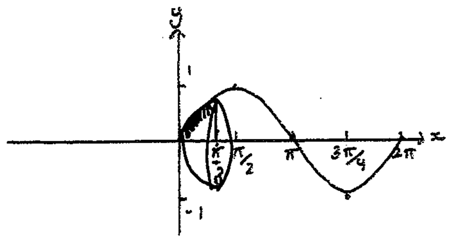
$\therefore t = 9, -1$

(B)

6. ${}^{15}C_3 \times {}^{10}C_2$

$= 20475$ (D)

7.



$$V = \pi \int_0^{\frac{\pi}{3}} \sin^2 x \; dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{3}} (1 - \cos 2x)\, dx$$

$$= \frac{\pi}{2} \left[x - \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{3}}$$

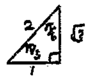
$$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3} - \left(0 - \frac{1}{2}\sin(0)\right) \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{1}{2}\sin\frac{\pi}{3} \right]$$

$$= \frac{\pi^2}{6} - \frac{\pi}{4} \times \frac{\sqrt{3}}{2}$$

$$= \left(\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}\right) u^3$$ (B)

$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = (1 - \sin^2 x) - \sin^2 x$
$\cos 2x = 1 - 2\sin^2 x$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$



8. A. As $x \to 0$ $\frac{dy}{dx} \to 0$

B. As $x \to 0$ $\frac{dy}{dx} \to y$ ×

C. As $x \to 0$ $\frac{dy}{dx} \to \infty$ ×

D As $x \to 0$ $\frac{dy}{dx} \to 0$

∴ A or D

A. As $y \to 0$ $\frac{dy}{dx} \to 0$ ×

D. As $y \to 0$ $\frac{dy}{dx} \to \infty$ (D)

$x = 0$, $\frac{dy}{dx} = \frac{1}{y}$ and y will determine the sign

9. $T = 185 - Pe^{-kt}$

$t = 2$

$$\frac{dT}{dt} = Pke^{-2k}$$

$$0.76\cancel{Pk} = \cancel{Pk}e^{-2k}$$

$$e^{-2k} = 0.76$$

$$-2k = \ln 0.76$$

$$k = \frac{\ln 0.76}{-2}$$

$$k = 0.137$$ (C)

$\frac{dT}{dt} = Pke^{-kt}$

$t = 0$ $\frac{dT}{dt} = Pke^{-k \times 0}$

$\frac{dT}{dt} = Pk$

10. $80 - (12 + 11)$

$= 57$

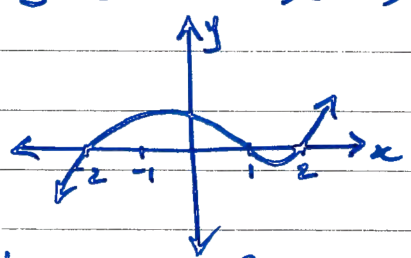
$57 \div 3 = 19$ Cannot have 19 of each

∴ $19 + 1 = 20$ white cars ∴ (D)

MATHEMATICS EXTENSION 1 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\cos 2\theta = 2\cos^2\theta - 1$		
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$		
$\int \cos^2\left(\frac{x}{6}\right) dx = \frac{1}{2}\int \left(1+\cos 2\left(\frac{x}{6}\right)\right) dx$	1	1 Mark for changing the equation correctly
$= \frac{1}{2}\int \left(1+\cos\left(\frac{x}{3}\right)\right) dx$		
$= \frac{1}{2}\left(x + 3\sin\left(\frac{x}{3}\right)\right) + c$		
$= \frac{x}{2} + \frac{3}{2}\sin\left(\frac{x}{3}\right) + c$	1	1 mark for correct integral
Most students did the change of equation correctly but had difficulty with the correct integral.		
b) $(1+2x)(2+x)^{12}$		
$= (1+2x)\left({}^{12}C_0 2^{12} + {}^{12}C_1 2^{11} x + {}^{12}C_2 2^{10} x^2 + \ldots\right.$	1	1 mark for the expansion of $(2+x)^{12}$
$\left.\ldots + {}^{12}C_8 2^4 x^8 + {}^{12}C_9 2^3 x^9 + \ldots + {}^{12}C_{12} x^{12}\right)$		
$= {}^{12}C_0 2^{12} + {}^{12}C_1 2^{11} x + {}^{12}C_2 x^2 + \ldots + {}^{12}C_9 2^3 x^9 + \ldots$		
$\ldots + {}^{12}C_{12} x^{12} + \ldots + {}^{12}C_0 2^{12} \times 2x + {}^{12}C_1 2^{11} \times 2x + \ldots$		
$\ldots + {}^{12}C_8 2^4 x^8 \times 2x + \ldots + {}^{12}C_{12} x^{12} \times 2x$		
$= {}^{12}C_0 2^{12} + {}^{12}C_1 2^{11} x + \ldots + {}^{12}C_9 2^3 x^9 + \ldots + {}^{12}C_8 2^5 x^9 + \ldots$		
$\ldots + {}^{12}C_{12} x^{13} \times 2$		
Terms containing x^9 are		
${}^{12}C_9 2^3 x^9 + {}^{12}C_8 2^5 x^9$	1	1 mark for the terms containing x^9
$= \left({}^{12}C_9 2^3 + {}^{12}C_8 2^5\right) x^9$		
Coefficient of x^9		
${}^{12}C_9 2^3 + {}^{12}C_8 2^5$		
$= 17600$	1	1 mark for the correct answer
Most students had difficulty with this question.		

MATHEMATICS EXTENSION 1 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) —, V, —, V, —, V, —, —		
Vowels are E, O, E		
ie. 3 vowels with 2 the same		
Consonants are G, M, T, R, Y		
ie. 5 consonants with no repeats		
Vowels can be placed in $\frac{3!}{2!}$ ways		
$= 3$ ways	1	0 mark for
Consonants can be placed in $5!$ ways		finding either
Total ways $= 3 \times 5!$	1	vowels or
$= 360$ ways		consonants.
Most students did this well but did not take into account that two of the vowels were the same.		
d) $\frac{x^2}{x-2} \geq -1 \quad (x \neq 2)$		
$\frac{x^2(x-2)^2}{(x-2)} \geq -1(x-2)^2$		
$x^2(x-2) \geq -(x-2)^2$		
$x^2(x-2)+(x-2)^2 \geq 0$		
$(x-2)(x^2+x-2) \geq 0$		
$(x-2)(x+2)(x-1) \geq 0$	1	1 mark for getting the equality ≥ 0
graph of $y=(x-2)(x+2)(x-1)$		
	$\frac{1}{2}$	$\frac{1}{2}$ mark for correct factorisation
	$\frac{1}{2}$	$\frac{1}{2}$ mark for use of correct graph
Graph is above the x axis for		
$-2 \leq x \leq 1$ and $x \geq 2$	1	for correct answer
but $x \neq 2$ $\therefore$ $-2 \leq x \leq 1$, $x > 2$	$-\frac{1}{2}$	if $x \geq 2$

MATHEMATICS EXTENSION 1 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

d) Alternative method

$$\frac{x^2}{x-2} \geq -1$$

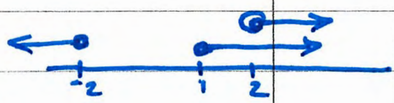
Take $x > 2$

$$x^2 \geq -(x-2)$$

$$x^2 + x - 2 \geq 0$$

$$(x+2)(x-1) \geq 0$$

$$x \leq -2, \quad x \geq 1$$



For both to be true $x > 2$

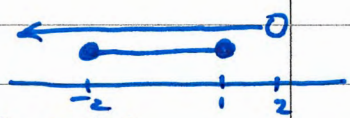
Take $x < 2$

$$x^2 \leq -(x-2)$$

$$x^2 + x - 2 \leq 0$$

$$(x+2)(x-1) \leq 0$$

$$x \geq -2, \quad x \leq 1$$

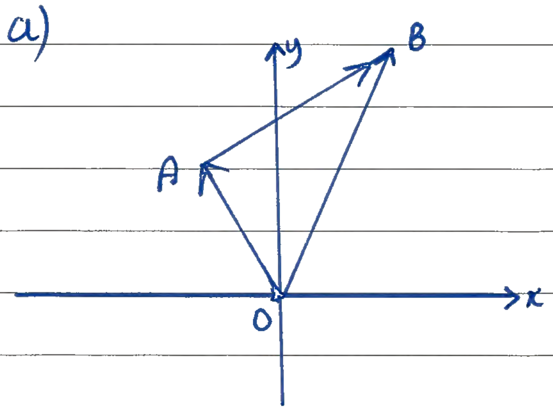


For both to be true $x \geq -2, x \leq 1$

$\therefore$ solution

$$-2 \leq x \leq 1, \quad x > 2$$

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)  $\vec{AB} = \vec{OB} - \vec{OA}$		This question was very well done.
$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}$		— ½ mark
$= \begin{pmatrix} 3 \\ 2 \end{pmatrix}$		— ½ mark
		1
b) $\int_0^1 \frac{t}{\sqrt{1+t}}\,dt = I$		This question was not done very well by many students.
$t = u^2 - 1$ $\frac{dt}{du} = 2u$ $dt = 2u\,du$		
Also $u^2 = t+1$ when $t=1$, $u^2 = 1+1 = 2$ $u = \sqrt{2}$ $t=0$, $u^2 = 1$ $u = 1$		Many students did not change the limits of integration —1 mark for correct limits
$I = \int_1^{\sqrt{2}} \frac{u^2-1}{\sqrt{1+u^2-1}} \times 2u\,du$		
$= \int_1^{\sqrt{2}} \frac{u^2-1}{\cancel{u}} \times 2\cancel{u}\,du$		
$= \int_1^{\sqrt{2}} 2(u^2-1)\,du$ 1 mark		—1 mark for correct integrand
$= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$		— ½ mark for integrating correctly

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) cont'd.		
$= 2\left[\frac{(\sqrt{2})^3}{3} - \sqrt{2} - \left(\frac{1}{3} - 1\right)\right]$		
$= 2\left[\frac{2\sqrt{2}}{3} - \sqrt{2} - \frac{1}{3} + 1\right]$		
$= 2\left[\frac{2\sqrt{2}}{3} - \sqrt{2} + \frac{2}{3}\right]$		$-\frac{1}{2}$ mark for substituting and expanding correctly.
$= 2\left[\frac{2\sqrt{3} - 3\sqrt{2} + 2}{3}\right]$		
$= \frac{2(2 - \sqrt{2})}{3}$		
$= \frac{4 - 2\sqrt{2}}{3}$		3
Note: Many students wrote their integrand in terms of two different variables such as t and u and this is NOT CORRECT.		
Many students did this: $\int_0^1 \frac{t}{\sqrt{1+t}}\,dt$		
These are the limits of integration for t not u, yet the integrand is in terms of u $\quad = \int_0^1 \frac{u^2-1}{u}\,du$		

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $P(x) = x^3 + ax^2 + bx - 12$		
$P'(x) = 3x^2 + 2ax + b$	½	Not many students used this method.
If $x = 2$ is a double root then		
$P(2) = P'(2) = 0$		Students should revise the Multiple Root theorem for the HSC.
For $P(2) = 0$		
$2^3 + a(2)^2 + b(2) - 12 = 0$		
$8 + 4a + 2b - 12 = 0$		
$4a + 2b = 4$		
$2a + b = 2$ --- (1)	½	
For $P'(2) = 0$		
$3(2)^2 + 2a(2) + b = 0$	½	
$12 + 4a + b = 0$		
$4a + b = -12$ --- (2)	½	
Solving simultaneously (1) – (2)		
$-2a = 14$		
$\therefore a = -7$ sub in (1)	½	
$2(-7) + b = 2$		
$b = 2 + 14$		
$b = 16$	½	
		3

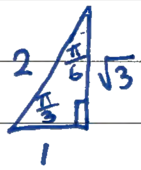
MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) Alternate method 1		
If $x=2$ is a double root then the roots of the cubic are:		
$2, 2, \gamma$.		
Sum of roots, $\Sigma\alpha = \frac{-b}{a}$		
$2+2+\gamma=-a$		
$4+\gamma=-a$ ---- (1)		
Sum of the roots 2 at a time, $\Sigma\alpha\beta = \frac{c}{a}$		
$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$		
$4+2\gamma+2\gamma=b$		
$4+4\gamma=b$ -- (2)	1	
Sum of the roots 3 at a time, $\Sigma\alpha\beta\gamma = \frac{-d}{a}$		
$2\times2\times\gamma=12$		
$4\gamma=12$	1	
$\gamma=3$		
sub in (1), $4+3=-a$		
$a=-7$	½	
sub in (2) $4+4\times3=b$		
$b=16$	½	

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) Alternate Method 2		
If $x=2$ is a double root of $P(x)$		
then		
$P(x) = (x-2)^2(x-\gamma)$		
$P(x) = (x^2-4x+4)(x-\gamma)$ --- (1)	1	
Equating constants		
$-4\gamma = -12$		
$\gamma = 3$ sub in (1)	½	
$P(x) = (x^2-4x+4)(x-3)$	½	
$= x^3 - 3x^2 - 4x^2 + 12x + 4x - 12$		
$= x^3 - 7x^2 + 16x - 12$		
$\therefore a = -7$ and $b = 16$	1	
		3

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) $\sqrt{3}\sin\theta - \cos\theta = R\sin(\theta - \alpha)$		
$\sqrt{3}\sin\theta - \cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$		
Equating parts		
$R\cos\alpha = \sqrt{3}$ --- ① and $R\sin\alpha = 1$ --- ②		
①^2+②^2 $R^2\sin^2\alpha + R^2\cos^2\alpha = 1+3$		
$R^2(\sin^2\alpha+\cos^2\alpha) = 4$		
$R^2 = 4$		
$R = 2$, $R>0$		— ½ mark
$\cos\alpha = \frac{\sqrt{3}}{2}$ and $\sin\alpha = \frac{1}{2}$		
$\therefore \alpha$ is in 1st Quad		
 $\therefore \frac{\sin\alpha}{\cos\alpha} = \frac{1/2}{\sqrt{3}/2}$		
$\tan\alpha = \frac{1}{\sqrt{3}}$		— ½ mark
$\therefore \alpha = \frac{\pi}{6}$		— ½ mark
$\therefore \sqrt{3}\sin\theta - \cos\theta = 2\sin(\theta - \frac{\pi}{6})$		
Now		
$\sqrt{3}\sin\theta - \cos\theta = 1$		
i.e $2\sin(\theta - \frac{\pi}{6}) = 1$		
$\sin(\theta - \frac{\pi}{6}) = \frac{1}{2}$ If $0 \leq \theta \leq 2\pi$ $-\frac{\pi}{6} \leq \theta - \frac{\pi}{6} \leq 2\pi - \frac{\pi}{6}$ $-\frac{\pi}{6} \leq \theta - \frac{\pi}{6} \leq \frac{11\pi}{6}$		— ½ mark
$\therefore \theta - \frac{\pi}{6} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$		
$= \frac{\pi}{6}, \frac{5\pi}{6}$		— ½ mark
and		
$\theta = \frac{\pi}{3}, \pi$		— ½ mark 3

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) Cont'd		
Note: Many students were able to to find $R=2$, but many students could not work out which quadrant α was in and thus did not find α successfully.		
Please review "auxiliary angle" method		

MATHEMATICS EXTENSION 1 – QUESTION 13

a) $\frac{dy}{dx} = e^x \cos^2 y$

$\therefore \frac{1}{\cos^2 y} dy = e^x dx$

$\sec^2 y\, dy = e^x dx$

$\int \sec^2 y\, dy = \int e^x dx$

$\tan y = e^x + C$

when $x = 2$, $y = 0$

$\therefore \tan 0 = e^2 + C$

$C = -e^2$

$\therefore \tan y = e^x - e^2$

$y = \tan^{-1}(e^x - e^2)$

3 marks Complete solution, including integration and finding C

2 marks Correct simplification of integrand using $\frac{1}{\cos\theta} = \sec\theta$

1 mark Correct separation of variables

b) Let the length of the cube be x (Don't use 'l' as a variable for length)

$\therefore V = x^3$ $A = 6x^2$

$\frac{dV}{dx} = 3x^2$ $\frac{dA}{dx} = 12x$ Also, $\frac{dV}{dt} = 5$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dV} \times \frac{dV}{dt}$

$= 12x \times \frac{1}{3x^2} \times 5$

$= \frac{20}{x}$

when $x = 600\text{mm}$, $\frac{dA}{dt} = \frac{20}{600}$ (Take care with units)

$= \frac{1}{30}\ \text{mm}^2\text{s}^{-1}$

3 marks Complete solution

2 marks Correctly combining 2 or more rates in a useful way.

1 mark Correct $\frac{dV}{dx}$ or $\frac{dA}{dx}$

Note that $V = 100A$ only at this instant; it is not useful when finding a general rate of change.

c) i $y = \sin^{-1} \frac{1}{x}$

$$y' = \frac{\frac{d}{dx}\left(\frac{1}{x}\right)}{\sqrt{1 - \left(\frac{1}{x}\right)^2}}$$ (from the reference sheet)

$$= \frac{-x^{-2}}{\sqrt{1 - \frac{1}{x^2}}}$$

$$= \frac{-1}{x^2\sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}}}$$

$$= \frac{-1}{x^2\sqrt{\frac{1}{x^2}(x^2-1)}}$$

$$= \frac{-1}{x^2\sqrt{\left(\frac{1}{x}\right)^2}\sqrt{x^2-1}}$$

This is a "show" question: you must show every step. Don't leave anything out.

$$= \frac{-1}{x^2 \times \frac{1}{x} \times \sqrt{x^2-1}}$$ (for $x > 0$)

$$= \frac{-1}{x\sqrt{x^2-1}}$$

2 marks Complete solution
1 mark Correct use of formula sheet

An important word on notation:
while $\frac{d}{dx}(y)$, $\frac{dy}{dx}$, and y' are all well-defined ways of expressing the derivative of y with respect to x, $\left(\frac{1}{x}\right)'$ DOES NOT mean the same thing as $\frac{d}{dx}\left(\frac{1}{x}\right)$.

MATHEMATICS EXTENSION 1 – QUESTION 13

c ii when $x = 2$, $y = \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6}$

$y' = \frac{-1}{2\sqrt{2^2-1}}$

$= \frac{-1}{2\sqrt{3}}$

$\therefore y - \frac{\pi}{6} = \frac{-1}{2\sqrt{3}}(x-2)$

$2\sqrt{3}y - \frac{\pi\sqrt{3}}{3} = -x+2$

$x + 2\sqrt{3}y - 2 - \frac{\pi\sqrt{3}}{3} = 0$

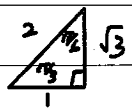
2 marks Complete solution

1 mark Correct gradient

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Step 1 - Base Case - prove true for $n=1$		
For $n=1$ $\quad 10^n + 3\times 4^{n+2} + 5$		
$= 10^1 + 3\times 4^{1+2} + 5$		
$= 10 + 192 + 5$		
$= 207$		
$= 9\times 23$ Which is divisible by 9	½ -	Correct proof to prove true
$\therefore$ true for $n=1$.		for $n=1$
Step 2 - Inductive hypothesis - assume true or $n=k$		
ie $10^k + 3\times 4^{k+2} + 5 = 9M$ ie $10^k = 9M - 3\times 4^{k+2} - 5$		
where $M \in \mathbb{Z}$	½	stating $M \in \mathbb{Z}$
step 3 - Inductive step - Prove true for $n=k+1$		
Prove $10^{k+1} + 3\times 4^{k+3} + 5$ is divisible by 9.		
$10^k\cdot 10 + 3\cdot 4^{k+3} + 5$		
$= 10(9M - 3\times 4^{k+2} - 5) + 3\cdot 4^{k+3} + 5$ By the assumption.	1	using the correct
$= 90M - 30\times 4^{k+2} - 50 + 3\cdot 4^{k+3} + 5$		form of the assumption
$= 90M - 30\times 4^k\cdot 4^2 - 50 + 3\cdot 4^k\cdot 4^3 + 5$		
$= 90M - 480\cdot 4^k - 45 + 192\cdot 4^k$		Many students tried
$= 90M - 288\cdot 4^k - 45$		to only use $3\cdot 4^{k+1} = 9M - 10^k - 5$
$= 9(10M - 32\cdot 4^k - 5)$		which was unsuccessful.
Which is divisible by 9.	1	Correct proof including conclusion (−½ if no conclusion with correct proof)
$\therefore$ true for $n=k+1$, since true for $n=k$		
$\therefore$ By the principle of mathematical induction it is true for all integer $n\geq 1$.		

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$ $x = \cos\theta$, $\frac{dx}{d\theta} = -\sin\theta$, $dx = -\sin\theta\,d\theta$ when $x = 1$: $1 = \cos\theta$ $\therefore \theta = 0$ when $x = \frac{1}{2}$: $\frac{1}{2} = \cos\theta$, $\theta = \frac{\pi}{3}$.		
$= \int_{\frac{\pi}{3}}^{0} \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta} \times -\sin\theta\,d\theta$ 1		1 forms correct integral with correct bounds
$= \int_{\frac{\pi}{3}}^{0} \frac{\sin\theta}{\cos^2\theta} \times -\sin\theta\,d\theta$ 		
$= \int_{\frac{\pi}{3}}^{0} -\frac{\sin^2\theta}{\cos^2\theta}\,d\theta$ 1 OR		1 simplifying to the step given.
$= \int_{\frac{\pi}{3}}^{0} -\tan^2\theta\,d\theta$ 1 $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$		
$= -\int_{\frac{\pi}{3}}^{0} (\sec^2\theta - 1)\,d\theta$		
$= \int_{\frac{\pi}{3}}^{0} (1 - \sec^2\theta)\,d\theta$		
$= \left[\theta - \tan\theta\right]_{\frac{\pi}{3}}^{0}$		
$= (0 - \tan 0) - \left(\frac{\pi}{3} - \tan\frac{\pi}{3}\right)$		
$= -\frac{\pi}{3} + \sqrt{3}$		
$= \sqrt{3} - \frac{\pi}{3}$ 1	1	Correct answer

MATHEMATICS EXTENSION 1 – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$y = \frac{x^2}{2} + 2$ $\quad y = \frac{20}{\pi}\sin^{-1}\frac{x}{4}$		
$2y = x^2 + 4$ $\quad \frac{\pi}{20}y = \sin^{-1}\frac{x}{4}$ $\quad -\frac{\pi}{2} \le \frac{x}{4} \le \frac{\pi}{2}$, $-2\pi \le x \le 2\pi$		
$x^2 = 2y - 4$ $\quad \sin\frac{\pi}{20}y = \frac{x}{4}$		
$x = 4\sin\frac{\pi}{20}y$		
$x^2 = 16\sin^2\frac{\pi}{20}y$		
when $x = 0$ $\quad x = 4$		
$y = \frac{0^2}{2} + 2$ $\quad y = \frac{16}{2} + 2$		
$y = 2$ $\quad = 10$		
and $y = \frac{20}{\pi}\sin^{-1}0$ $\quad y = \frac{20}{\pi}\sin^{-1}\frac{4}{4}$	1	Correct integral ½
$y = 0$ $\quad y = \frac{20}{\pi} \times \frac{\pi}{2}$		Correct bounds ½
(1) $\quad y = 10$ (1)	1	Correct integral ½
$V = \pi\int_0^{10} 16\sin^2\frac{\pi}{20}y \, dy - \pi\int_2^{10}(2y-4)\,dy$		Correct bounds ½
$= \frac{16\pi}{2}\int_0^{10}\left(1 - \cos\frac{\pi}{10}y\right)dy - \pi\left[y^2 - 4y\right]_2^{10}$ (1) —		Correct use of $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$
$= 8\pi\left[y - \frac{10}{\pi}\sin\frac{\pi}{10}y\right]_0^{10} - \pi\left[(100 - 40) - (4 - 8)\right]$		
$= 8\pi\left[10 - \frac{10}{\pi}\sin\pi - \left(0 - \frac{10}{\pi}\sin 0\right)\right] - \pi(60 + 4)$		
$= 8\pi\left[10 - 0\right] - 64\pi$		
$= (80 - 64)\pi$		
$= 16\pi\ u^3$ (1)	1	Correct answer

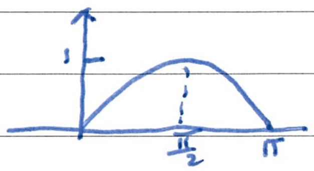
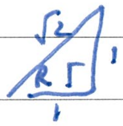
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - \sin^2\theta - \sin^2\theta$

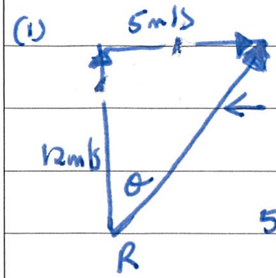
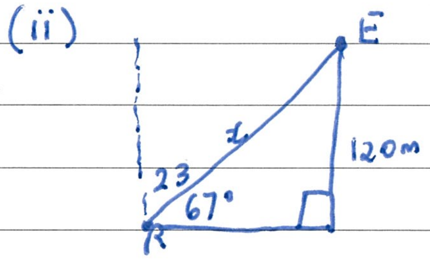
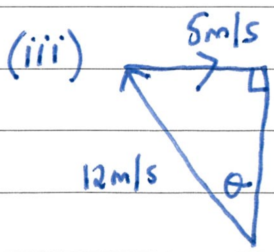
$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\therefore \sin^2\frac{\pi}{20}y = \frac{1}{2}\left(1 - \cos 2 \times \frac{\pi}{20}y\right)$

MATHEMATICS EXTENSION 1 – QUESTION 15 HSC trial 2022		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
Q15 a (4 marks)		
Solve $\sin 2x = \tan x$ for $0 \le x \le \pi$		
Method 1		
$2\sin x \cos x = \tan x$ $\quad x \neq \frac{\pi}{2}$.		
$2\sin x \cos x = \dfrac{\sin x}{\cos x}$	1	NB please DON'T DIVIDE by $\sin x$.
$2\sin x \cos^2 x = \sin x$		this was a significant issue for many students.
$2\sin x \cos^2 x - \sin x = 0$		It meant $\sin x = 0$ was not considered.
$\sin x\,(2\cos^2 x - 1) = 0$ or $\sin x(1 - 2\sin^2 x) = 0$	1	
$\therefore \sin x = 0$ or $2\cos^2 x - 1 = 0$		
 $\quad \cos^2 x = \frac{1}{2}$, $\quad \cos^2 x = {}^{+}\frac{1}{\sqrt{2}}$	2	
solutions in both quadrants. $\cos R = \frac{1}{\sqrt{2}}$		
$x = 0$ or π $\quad$  $\quad R = \frac{\pi}{4}$		
$\therefore x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$		
$(0 \le x \le \pi)$		
$\therefore x = 0, \frac{\pi}{4}, \frac{3\pi}{4}$ or π		
$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$		
Method 2. t formula.		
let $t = \tan x$		
then		
$\sin 2x = \dfrac{2t}{1+t^2}$ $\quad$ check where $\tan x$ is undefined		
$\dfrac{2t}{1+t^2} = t$	1	Similarly, don't divide by t.
$2t = t + t^3$ $\quad t = 0, 1$ or -1	1	
$t^3 - t = 0$ $\quad \tan x = 0, \tan x = 1, \tan x = -1$		
$t(t^2 - 1) = 0$ $\quad x = 0, \pi/4, \frac{3\pi}{4}$, or π	2	
$t(t-1)(t+1)$ $\quad \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$.		

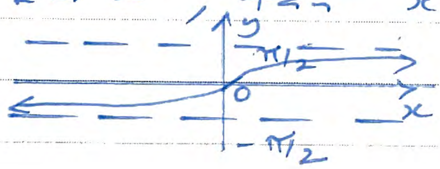
MATHEMATICS EXTENSION 1 – QUESTION 15 (b)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(i)  resultant vector.		No part marks
$5\underset{\sim}{i} + 12j$	1	mark
$\lvert v\rvert = \sqrt{5^2+12^2}$		
$= \sqrt{169}$		
$= 13$ m/s	1	mark
		suprisingly; many students struggled to draw the diagram
(ii) 		
$\tan\theta = \frac{5}{12}$		
$\theta = 023°T$	1	
gave $\frac{1}{2}$ if NO 0		
$\sin 67° = \frac{120}{RE}$		
$RE = \frac{120}{\sin 67°}$		
$RE \doteqdot 130$m	1	
(iii) 		
$\sin\theta = \frac{5}{12}$	1	
$\theta = 24°37'$ $\frac{1}{2}$		
$360° - 24°37'$		
$\doteqdot 335°T$		

MATHEMATICS EXTENSION 1 – QUESTION 16 a

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
The trivial solutions are $P=0$ and $P=M$ ($\frac{1}{2}$mk, $\frac{1}{2}$mk)		Only 1 student in the entire cohort remembered about the trivial solution.
$\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right)$		
$\frac{dP}{dt} = kP\left(\frac{M-P}{M}\right)$		
$\frac{dt}{dP} = \frac{1}{kP} \times \frac{M}{M-P}$		Also, generally from
$k\,dt = \frac{M}{P(M-P)}\,dP$		
$\int\left(\frac{1}{P}+\frac{1}{M-P}\right)dP = \int k\,dt$	1mk	$P = \frac{MAe^{kt}}{1+Ae^{kt}}$
(The process of substitution should be very explicitly displayed)		students should have divided both the numerator and denominator by Ae^{kt} to obtain a much neater looking expression.
$\ln\lvert P\rvert - \ln\lvert M-P\rvert = kt + c$		
$\ln\left\lvert\frac{P}{M-P}\right\rvert = kt + c$ — $\frac{1}{2}$mk		
$\left\lvert\frac{P}{M-P}\right\rvert = e^{kt+c}$		
$\left\lvert\frac{P}{M-P}\right\rvert = e^{kt}\cdot e^{c}$		
$= Ae^{kt}$, where $A = e^{c}$ is a positive constant.		Note that $e^{c} > 0$
$\frac{P}{M-P} = Ae^{kt}$, where A is any constant now.		— $\frac{1}{2}$mk
$P = MAe^{kt} - PAe^{kt}$		
$P + PAe^{kt} = MAe^{kt}$		
$P(1+Ae^{kt}) = MAe^{kt}$		
$\therefore P = \frac{MAe^{kt}}{1+Ae^{kt}}$ $(\because Ae^{kt} \neq 0)$		— 1mk
$P = \frac{M}{\frac{1}{A}e^{-kt}+1}$ where $B = \frac{1}{A}$, $A \neq 0$		
$\therefore P = \frac{M}{Be^{-kt}+1}$ where $B \neq 0$, $P=0$, $P=M$		

MATHEMATICS EXTENSION 1 – QUESTION 16 b

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b i) Greatest slope at $x=0$	$\frac{1}{2}$ mk	A number of students just wrote down $x=0$ so lost $\frac{1}{2}$ mk
However, the question is asking you to write down the point at which the greatest slope occurs. Therefore, substitute $x=0$ into the expression for $f(x) = 2\tan^{-1}x$		
$f(0) = 2 \times \tan^{-1}(0)$ $= 0$		
$\therefore$ point $= (0,0)$	$\frac{1}{2}$ mk	for the corresponding y-coordinate.
ii) $\int_0^k f'(x)\,dx$		
$\int_0^k \frac{2}{1+x^2}\,dx$ or $\left[2\tan^{-1}x\right]_0^k$	1 mk	
(As differentiation and integration are inverse operations of each other)		
iii) $A = 2 \times \int_0^k \frac{2}{1+x^2}\,dx$		
$= 4\left[\tan^{-1}x\right]_0^k$	$\frac{1}{2}$ mk	
$= 4\left[\tan^{-1}k - \tan^{-1}0\right]$		
To find the area under this curve as $k \to \infty$: As $k \to \infty$, $\tan^{-1}x \to \pi/2$  $= 4 \times (\pi/2 - 0)$ as $k \to \infty$, $\tan^{-1}x \to \pi/2$	1 mk	

$= 4 \times \pi/2$

$= 2\pi$ $\frac{1}{2}$ mk

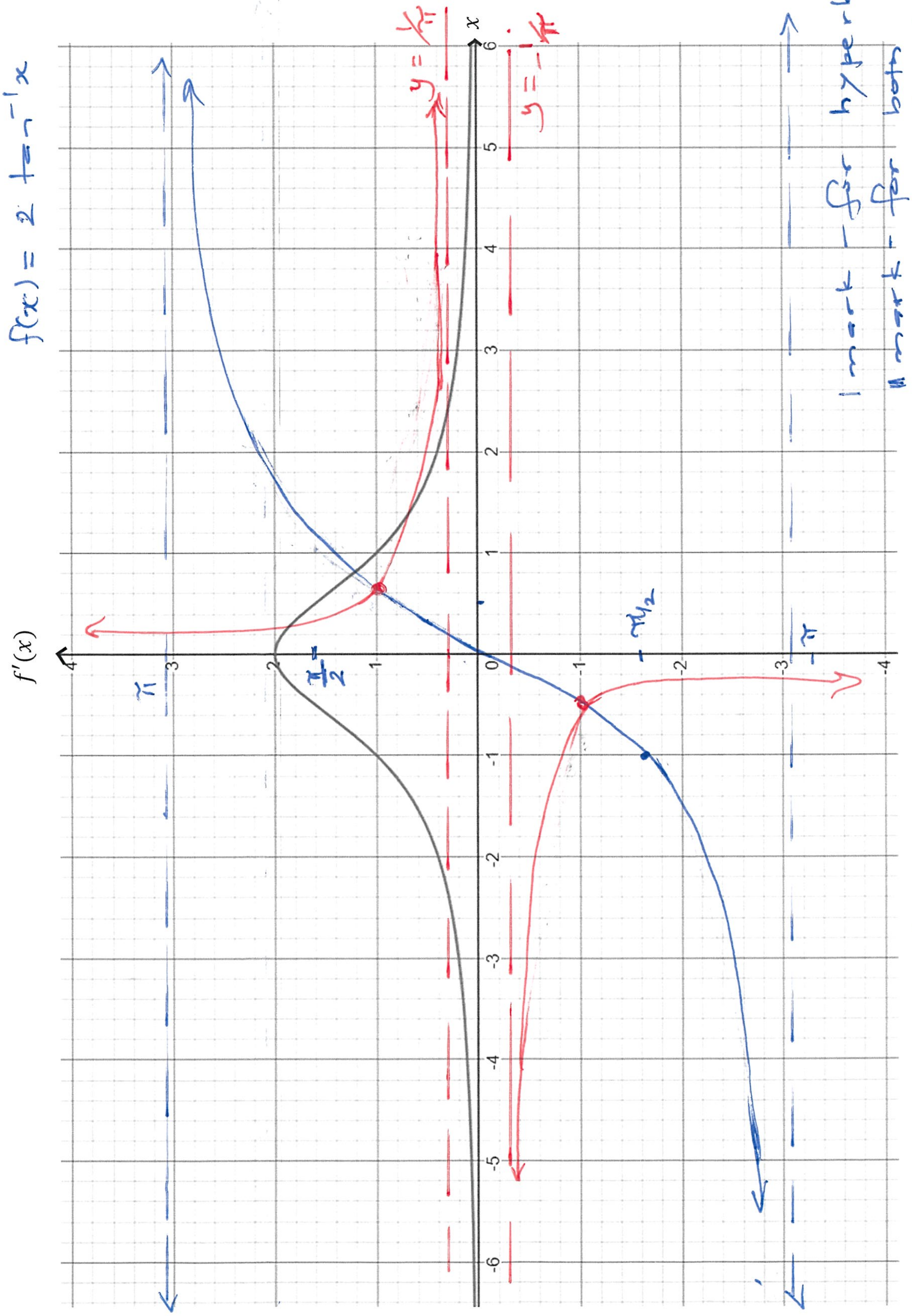
Student Number:
Teacher:

QUESTION 16 (b) (iv)

Marks

The graph below shows the **derivative** of $f(x) = 2\tan^{-1}x$.

On this diagram, sketch the graph of $y = \dfrac{1}{f(x)}$. **2**



1 mark – for hyperbola – correct quadrants

1 mark – for both the horizontal and vertical asymptotes

Poorly attempted as only a few students drew the horizontal asymptotes accurately

PLEASE PLACE THIS SHEET IN YOUR ANSWER BOOKLET FOR QUESTION 16.